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Performance of an endoreversible four-heat-reservoir absorption heat pump with a generalized heat transfer law ☆

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Abstract

An endoreversible four-heat-reservoir absorption heat pump cycle model with a generalized heat transfer law $Q \propto \Delta(T^n)$ is established. The general relation between the coefficient of performance (COP) and the heating load with $Q \propto \Delta(T^n)$ is deduced. The fundamental optimal relation, the optimal temperatures of working substance, as well as the optimal heat transfer surface area distributions with linear phenomenological heat transfer law are derived. Moreover, the effects of heat transfer law on the performance of absorption heat pump are analyzed and the performance comparison before and after optimizing the distribution of the total heat transfer surface area is performed by numerical example.

Keywords: Finite-time thermodynamics; Thermodynamic optimization absorption heat pump; Heat transfer law; Optimal performance

1. Introduction

Due to environmental pollution and continually rising conventional fuel prices, the thermodynamic cycle, which not only can utilize low temperature waste heat, solar energy or geothermic energy, but also can diminish the pollution to the surroundings, attracts more and more peoples' attention. Absorption heat pump is one of the promising devices, which can upgrade low-grade heat to higher temperature levels and is harmless to the surroundings. Nowadays, finite-time thermodynamics (or finite surface thermodynamics, or endoreversible thermodynamics, or entropy generation minimization) [1–11] plays a fundamental role to evaluate the optimal performance of any thermodynamic cycles. Some authors [12–20] have investigated the performance of absorption heat pump cycle using finite-time thermodynamics, and some new significant results for optimal design of absorption heat pumps have been obtained. Chen and Yan [12], Herold [13], Goktun [14] and Chen [15] analyzed the performance of the three-heat-reservoir heat pump cycle with the loss of heat resistance [12,13], with losses of heat resistance and internal irreversibility [14], and with losses of heat resistance, heat leakage and internal irreversibility [15] with linear (Newtonian) heat transfer law. Kodal et al. [16] analyzed the thermoeconomic performance of the three-heat-reservoir heat pump cycle. A three-heat-reservoir absorption heat pump cycle is a simplified model of the absorption heat pump that the temperature of the condenser is equal to that of the absorber, but a real absorption heat pump is not. Therefore, a four-heat-reservoir absorption heat pump cycle model is closer to a real absorption heat pump. Chen [17] and Chen et al. [18] analyzed the performance of the four-heat-reservoir absorption heat pump cycle with losses of heat resistance and internal irreversibility [17], and with losses of heat resistance, heat leakage and internal irreversibility [18] with linear (Newtonian) heat transfer law.

Real heat transfer between heat reservoirs and working substance do not always obey linear (Newtonian) heat transfer law. Chen et al. [19,20] analyzed the performance of the three-heat-reservoir endoreversible absorption heat pump cycle with linear phenomenological heat transfer law. Therefore, the aim of this paper is to establish the endoreversible four-heat-reservoir ab-

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Nomenclature

A	total heat transfer surface area	m^2
A_i $(i = 1, 2, 3, 4)$	heat transfer surface area of the ith heat exchanger	m^2
n	heat transfer exponent	
Q_i $(i = 1, 2, 3, 4)$	rate of heat transfer of ith heat reservoir	kW
T_i $(i = a, c, e, g)$	temperature of ith heat reservoir	K
T_i $(i = 1, 2, 3, 4)$	working substance temperature in ith heat exchanger	K
UA	total heat inventory	$kW \cdot K^{-1}$
U_i $(i = 1, 2, 3, 4)$	Heat transfer coefficient of ith heat exchanger	$kW \cdot m^{-2} K^{-1}$
$U_i A_i$ $(i = 1, 2, 3, 4)$	Heat conductance of ith heat exchanger	$kW \cdot K^{-1}$

Greek symbol

ψ	COP	
Π	Heating load	kW
ξ	distribution of the total rate of heat output between the absorber and the condenser	

Subscript

1–4	working substance process in heat exchanger
a	absorber
e	evaporator
m	maximum
UA	after optimizing the distribution of the total heat exchanger inventory
ψ	At maximum COP point
A	after optimizing the distribution of the total heat transfer surface area
c	condenser
g	generator
r	Reversible cycle
Π	At maximum heating load point

sorption heat pump cycle model with a generalized heat transfer law $Q \propto \Delta(T^n)$, and analyzes the performance of the four-heat-reservoir endoreversible absorption heat pump cycle. The generalized heat transfer law $q \propto \Delta(T^n)$ includes some special cases. When $n = 1$, the heat transfer obeys Newtonian law; when $n = -1$, the heat transfer obeys linear phenomenological law used in irreversible thermodynamics, the heat transfer coefficients in this case are the so-called kinetic coefficients by Callen [21], and they should be negatives; when $n = 2$, the heat transfer is applicable to radiation propagated along a one-dimensional transmission line [22], and the heat transfer coefficient in this case is equal to $\pi^2 k^2/(6h)$, where h is the Planck's constant and k is the Stefan–Boltzmann constant; when $n = 3$, the heat transfer is applicable to radiation propagated along a two-dimensional surface [22]; when $n = 4$, the heat transfer obeys radiative law if all the bodies are black, and the heat transfer coefficient in this case is related to the Stefan–Boltzmann constant. A similar work for four-heat-reservoir absorption refrigerator was performed by Zheng et al. [23] with heat transfer law of $Q \propto \Delta(T^{-1})$.

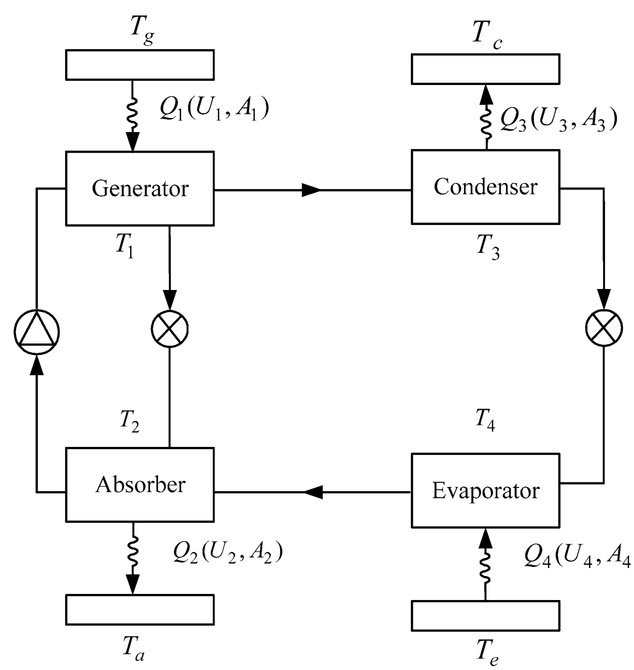

Fig. 1. An endoreversible four-heat-reservoir absorption heat pump cycle model.

2. Theoretical model

A four-heat-reservoir endoreversible absorption heat pump cycle consists of a generator, an absorber, a condenser and an evaporator, as shown in Fig. 1. Consider that the flow of the working substance in the closed cycle is steady closed flow fashion and the working substance exchanges heat with the heat reservoirs at temperature T_g, T_a, T_c and T_e in the generator, absorber, condenser and evaporator, respectively. The endoreversible absorption heat pump cycle means that the cycle with the sole loss of heat resistance between the working substance and the heat reservoirs and without any other irreversibilities inside the cycle and among the heat reservoirs. There are heat resistances between the working substance and the external heat reservoirs. Therefore, the temperatures of the working substance in the generator, absorber, condenser and evaporator are different from the heat reservoir temperatures and are T_1, T_2, T_3 and T_4, respectively, and the following equations hold: $T_g > T_1$, $T_a < T_2$, $T_c < T_3$ and $T_e > T_4$; the overall heat transfer coefficients in the generator, absorber, condenser and evaporator are U_1, U_2, U_3 and U_4, respectively; and the heat transfer surface areas of the generator, absorber, condenser and evaporator are A_1, A_2, A_3 and A_4, respectively. The work input required by the solution pump in the system is negligible relative to the energy input to the generator, and the work input is neglected for the purpose of analysis. It is assumed that the heat transfers obey a generalized heat transfer law of $Q \propto \Delta(T^n)$ ($n \neq 0$), where n is a heat transfer exponent. Then, the equations of the rates of heat transfer may be written as

$$
\begin{aligned}
Q_1 &= U_1A_1(T_g^n - T_1^n)\\
Q_2 &= U_2A_2(T_2^n - T_a^n)\\
Q_3 &= U_3A_3(T_3^n - T_c^n)\\
Q_4 &= U_4A_4(T_e^n - T_4^n)
\end{aligned}
\tag{1}
$$

where Q_1, Q_2, Q_3 and Q_4 are the rates of heat transfer in the generator, absorber, condenser and evaporator, respectively; U_i $(i = 1, 2, 3, 4) > 0$ when $n > 0$, and U_i and $(i = 1, 2, 3, 4) < 0$ when $n < 0$.

The original paper of Curzon and Ahlborn [1] discussed a model of reciprocating Carnot heat engine. In that condition, the heat transfers go through the same surface areas and different times. While the present work discussed a model of steady flow closed heat pump. In this condition, the heat transfers go through the same time and different surface areas. Therefore, Eq. (1) used the rate of heat transfer, and did not include the times of the processes or the cycle period.

From the first law of thermodynamics, one has $Q_2 + Q_3 - Q_1 - Q_4 = 0$. From the second law of thermodynamics and the endoreversible property of the cycle, one has $Q_2/T_2 + Q_3/T_3 - Q_1/T_1 - Q_4/T_4 = 0$. Defining a parameter $\xi = Q_3/Q_2$ that denotes the distribution of the total heat output rates between the absorber and the condenser.

According to the standard definitions of the COP (ψ) and the heating load (Π) of an absorption heat pump and Eq. (1), one obtains

$$
\begin{aligned}
\psi &= \frac{(1+\xi)Q_2}{Q_1} = \frac{(1+\xi)(T_1^{-1} - T_4^{-1})}{T_2^{-1} + \xi T_3^{-1} - (1+\xi)T_4^{-1}}\\
\Pi &= (1+\xi Q_2) = (1+\xi)U_2A_2(T_2^n - T_a^n)
\end{aligned}
\tag{2}
$$

According to Eq. (2), one has

$$
\begin{aligned}
Q_1 &= \Pi/\psi, \qquad Q_2 = \Pi/(1+\xi)\\
Q_3 &= \xi\Pi/(1+\xi), \qquad Q_4 = \Pi(\psi - 1)/\psi
\end{aligned}
\tag{3}
$$

According to Eqs. (1) and (3), one has

$$
\begin{aligned}
T_1 &= \left[T_g^n - \Pi/(U_1A_1\psi)\right]^{1/n}\\
T_2 &= \left\{T_a^n + \Pi/\left[(1+\xi)U_2A_2\right]\right\}^{1/n}\\
T_3 &= \left\{T_c^n + \xi\Pi/\left[(1+\xi)U_3A_3\right]\right\}^{1/n}\\
T_4 &= \left\{T_e^n - \Pi(\psi - 1)/(U_4A_4\psi)\right\}^{1/n}
\end{aligned}
\tag{4}
$$

Substituting Eq. (4) into the COP in Eq. (2) yields

$$
\begin{aligned}
&(1+\xi)\left[T_g^n - \Pi/(U_1A_1\psi)\right]^{-1/n}\\
&\quad - \psi\left\{T_a^n + \Pi/\left[(1+\xi)U_2A_2\right]\right\}^{-1/n}\\
&\quad - \xi\psi\left\{T_c^n + \xi\Pi/\left[(1+\xi)U_3A_3\right]\right\}^{-1/n}\\
&\quad + (1+\xi)(\psi - 1)\left[T_e^n - \Pi(\psi - 1)/(U_4A_4\psi)\right]^{-1/n} = 0
\end{aligned}
\tag{5}
$$

If $n = 1$, i.e. the heat transfers obey linear (Newtonian) law, according to Eq. (5), one has

$$
\begin{aligned}
&(1+\xi)\left[T_g - \Pi/(U_1A_1\psi)\right]^{-1} - \psi\left\{T_a + \Pi/\left[(1+\xi)U_2A_2\right]\right\}^{-1}\\
&\quad - \psi\left\{T_c/\xi + \Pi/\left[(1+\xi)U_3A_3\right]\right\}^{-1}\\
&\quad + (1+\xi)\left[T_e/(\psi - 1) - \Pi/(U_4A_4\psi)\right]^{-1} = 0
\end{aligned}
\tag{6}
$$

Eq. (6) is the general relation between the COP and the heating load of the endoreversible four-heat-reservoir absorption heat pump with linear (Newtonian) heat transfer law.

If $n = -1$, i.e. the heat transfers obey linear phenomenological law, according to Eq. (5), one has

$$
\begin{aligned}
&(1+\xi)\left[T_g^{-1} - \Pi/(U_1A_1\psi)\right] - \psi\left\{T_a^{-1} + \Pi/\left[(1+\xi)U_2A_2\right]\right\}\\
&\quad - \xi\psi\left\{T_c^{-1} + \xi\Pi/\left[(1+\xi)U_3A_3\right]\right\}\\
&\quad + (1+\xi)(\psi - 1)\left[T_e^{-1} - \Pi(\psi - 1)/(U_4A_4\psi)\right] = 0
\end{aligned}
\tag{7}
$$

Eq. (7) is the general relation between the COP and the heating load of the endoreversible four-heat-reservoir absorption heat pump with linear phenomenological heat transfer law.

3. Fundamental optimal relation between the COP and the heating load

The investment cost of an absorption heat pump mainly depends on the total heat transfer surface area A. Therefore, minimizing the total heat transfer surface area or optimizing the heat transfer surface area distributions is important to the optimal design of an absorption heat pump. For this reason, it makes sense to introduce the total heat transfer surface area A as a design constraint

$$
A = A_1 + A_2 + A_3 + A_4 \tag{8}
$$

For reciprocating cycle, to optimize the temperatures of working substance is equivalent to optimize the process times. While for steady flow cycle, to optimize the temperatures of working substance is equivalent to optimize the heat transfer surface area distributions. This conclusion has been proved for various two, three and four heat reservoir direct and inverse cycles. This means that the optimization for temperatures of working substance and the optimization for the heat transfer surface area distributions are twinning.

By optimizing the temperatures T_1, T_2, T_3 and T_4 of working substance, i.e. optimizing the heat transfer surface area distributions A_1/A, A_2/A, A_3/A and A_4/A, one can obtain the optimal COP for a given heating load and the optimal heating load for a given COP.

When $n = 1$, Refs. [17,18] have derived the fundamental optimal relation between the COP and the heating load.

When $n = -1$, this paper derives the fundamental optimal relation between the COP and the heating load as follows.

In the case of other heat transfer laws, it is difficult to derive the analytical fundamental optimal relation. One can only attain the optimal numerical solution by numerical optimization technology. When $n = 2$ and $n = 4$, next section of this paper depicts the fundamental optimal relation curves by numerical optimization technology.

According to Eqs. (1), (2), (8) and $n = -1$, the COP ψ_A and the heating load Π_A for the fixed total heat transfer surface area can be written as

$$\psi_A = \frac{(1+\xi)(1-x_1)}{1+\xi-x_2-\xi x_3} \tag{9}$$

$$\begin{aligned}\Pi_A = A(1+\xi)\bigg[&\frac{1+\xi-x_2-\xi x_3}{U_1(1-x_1)(x_1T_4^{-1}-T_g^{-1})} \\ &+\frac{1}{U_2(T_a^{-1}-x_2T_4^{-1})}+\frac{\xi}{U_3(T_c^{-1}-x_3T_4^{-1})} \\ &+\frac{x_2+\xi x_3-(1+\xi)x_1}{U_4(1-x_1)(T_4^{-1}-T_e^{-1})}\bigg]^{-1}\end{aligned} \tag{10}$$

where $x_1 = T_4/T_1$, $x_2 = T_4/T_2$ and $x_3 = T_4/T_3$.

To maximize the COP for a given heating load and maximize the heating load for a given COP, the Lagrangian function $L = \psi_A + \lambda\Pi_A$ or $L = \Pi_A + \lambda\psi_A$ is introduced, where λ is Lagrangian coefficient. Combining extremal conditions $\partial L/\partial x_1 = 0$, $\partial L/\partial x_2 = 0$, $\partial L/\partial x_3 = 0$ and $\partial L/\partial T_4 = 0$ simultaneously yields:

$$\begin{aligned}&U_1\big(x_1T_4^{-1}-T_g^{-1}\big) \\ &\quad = U_2\big(T_a^{-1}-x_2T_4^{-1}\big) = U_3\big(T_c^{-1}-x_3T_4^{-1}\big) \\ &\quad = U_4\big(T_4^{-1}-T_e^{-1}\big)\end{aligned} \tag{11}$$

According to Eqs. (9)–(11), one has

$$\begin{aligned}\Pi_A = {}& A(1+\xi)U_4\psi_A \\ &\times\frac{(1+\xi)(T_e^{-1}-T_g^{-1})+[T_a^{-1}+\xi T_c^{-1}-(1+\xi)T_e^{-1}]\psi_A}{[(1+\xi)(1-b_1)-(1+\xi+b_2+\xi b_3)\psi_A]^2}\end{aligned} \tag{12}$$

where $b_1 = (U_4/U_1)^{1/2}$, $b_2 = (U_4/U_2)^{1/2}$ and $b_3 = (U_4/U_3)^{1/2}$.

Eq. (12) determines the optimal heating load for a given COP, at the same time it determines the optimal COP for a given heating load of the endoreversible four-heat-reservoir absorption heat pump. Therefore, Eq. (12) is the fundamental optimal relation between the COP and the heating load of the endoreversible four-heat-reservoir absorption heat pump with linear phenomenological heat transfer law.

Likely, the relations between the optimal temperatures T_{1A}, T_{2A}, T_{3A}, and T_{4A} of working substance and the optimal COP are:

$$\begin{aligned}&T_{1A} = \frac{T_g}{1+b_1T_gu_A}, \qquad T_{2A} = \frac{T_a}{1-b_2T_au_A} \\ &T_{3A} = \frac{T_c}{1-b_3T_cu_A}, \qquad T_{4A} = \frac{T_e}{1+T_eu_A}\end{aligned} \tag{13}$$

where

$$u_A = \frac{(1+\xi)(T_g^{-1}-T_e^{-1})-[T_a^{-1}+\xi T_c^{-1}-(1+\xi)T_e^{-1}]\psi_A}{(1+\xi)(1-b_1)-(1+\xi+b_2+\xi b_3)\psi_A}$$

The relations between the optimal heat transfer surface area distributions and the optimal COP are

$$\begin{aligned}\frac{A_1}{A} &= \frac{b_1(1+\xi)}{(b_1-1)(1+\xi)+(1+\xi+b_2+\xi b_3)\psi_A} \\ \frac{A_2}{A} &= \frac{b_2\psi_A}{(b_1-1)(1+\xi)+(1+\xi+b_2+\xi b_3)\psi_A} \\ \frac{A_3}{A} &= \frac{\xi b_3\psi_A}{(b_1-1)(1+\xi)+(1+\xi+b_2+\xi b_3)\psi_A} \\ \frac{A_4}{A} &= \frac{(1+\xi)(\psi_A-1)}{(b_1-1)(1+\xi)+(1+\xi+b_2+\xi b_3)\psi_A}\end{aligned} \tag{14}$$

4. Numerical examples and analysis

In order to analyze the effects of the heat transfer law on the general relation between the COP and the heating load of the cycle, according to Eq. (5), Fig. 2 depicts the general relation curves for four heat transfer laws, i.e. $n = -1, 1, 2, 4$. In the calculation, $T_g = 420$ K, $T_a = 340$ K, $T_c = 350$ K, $T_e = 300$ K, $\zeta = 1.1$, $b_1 = 1.30$, $b_2 = 1.40$, $b_3 = 0.80$, $A_1/A = 0.27$, $A_2/A = 0.32$, $A_3/A = 0.16$ and $A_4/A = 0.25$ are set. It can be seen from Fig. 2 that the general relation curves between the COP and the heating load of an endoreversible four-heat-reservoir absorption heat pump are monotonous. Because the minimum COP of the heat pump is $\psi = 1$, the heating load attaints to its maximum Π_m when $\psi = 1$. Substituting $\psi = 1$ into Eq. (5) for $n = -1, 1, 2, 4$ yields the corresponding Π_m.

According to Eq. (5), when $\psi = \psi_r$ [8]

$$\psi_r = \frac{(1+\xi)(T_e^{-1}-T_g^{-1})}{(T_e^{-1}-T_a^{-1})+\xi(T_e^{-1}-T_c^{-1})} \tag{15}$$

$\Pi = 0$. ψ_r is the reversible COP of a four-heat-reservoir absorption heat pump. When $0 < \psi < \psi_r$, the COP and the heating load are influenced by heat transfer laws quantitatively; and the COP for a given heating load and the heating load for a given COP decrease as n increases.

In order to analyze the effects of the heat transfer law on the fundamental optimal relation between the COP and the heating load of the cycle, according to Eq. (12), the fundamental optimal relation curve is depicted for $n = -1$; according to Refs. [17,18], the fundamental optimal relation curve is depicted for $n = 1$; according to Eqs. (5) and (8), the heat transfer

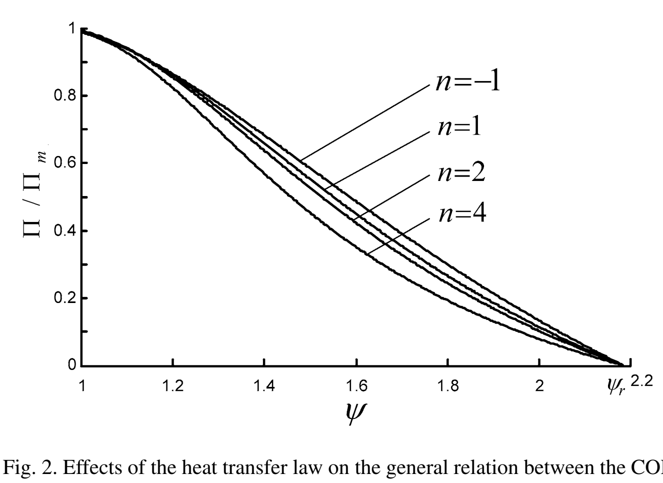

Fig. 2. Effects of the heat transfer law on the general relation between the COP and the heating load.

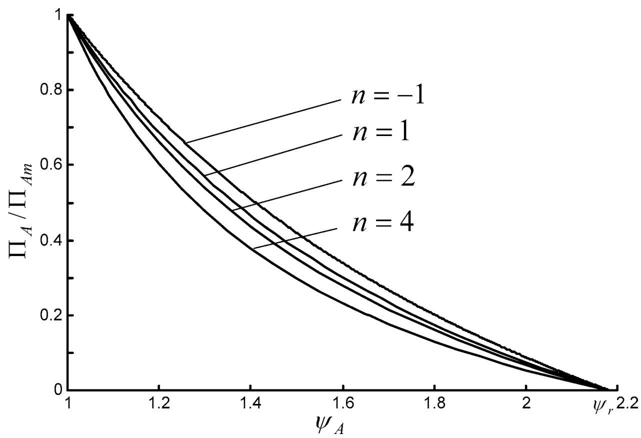

Fig. 3. Effects of the heat transfer law on the fundamental optimal relation between the COP and the heating load.

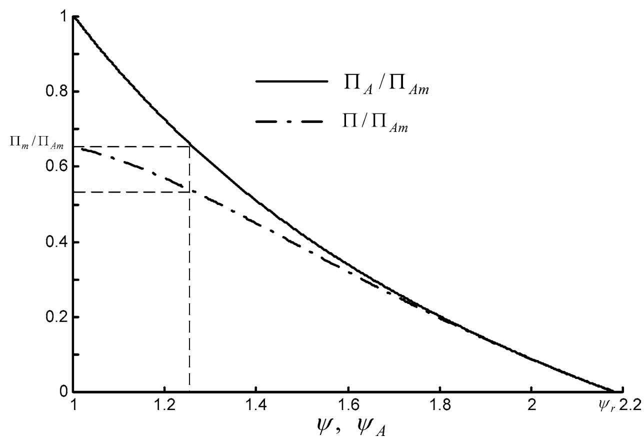

Fig. 4. Effects of the heat transfer surface area distributions on the cycle performance with $n=-1$.

surface area distributions are optimized by numerical optimization technology and the fundamental optimal relation curves are depicted for $n=2$ and $n=4$; as shown in Fig. 3. In the calculation, $T_g=420$ K, $T_a=340$ K, $T_c=350$ K, $T_e=300$ K, $\zeta=1.1$, $b_1=1.30$, $b_2=1.40$ and $b_3=0.80$ are set. It can be seen from Fig. 3 that the fundamental optimal relation curves between the COP and the heating load of an endoreversible four-heat-reservoir absorption heat pump are also monotonous. Fig. 3 shows the dimensionless heating load (the ratio of optimal heating load Π_A to the maximum heating load Π_{Am}) versus COP characteristics for four heat transfer laws. When $\psi_A=1$, the optimal heating load attaints to its maximum Π_{Am}. According to Eq. (12), when $\psi_A=\psi_r$, $\Pi_A=0$. When $0<\psi_A<\psi_r$, the optimal COP and the optimal heating load are influenced by heat transfer laws quantitatively. The absolute values of heating load for various heat transfer laws cannot be compared. However, one can see that both the optimal COP for a given dimensionless heating load and the optimal dimensionless heating load for a given COP decrease as n increases.

In order to analyze the effects of the heat transfer surface area distributions on the cycle performance, according to Eqs. (7) and (12), Fig. 4 depicts the $\Pi/\Pi_{Am}-\psi$ curve with fixed heat transfer surface area distributions and the $\Pi_A/\Pi_{Am}-\psi_A$ curve with the optimal heat transfer surface area distributions for $n=-1$. In the calculation, the values are the same as those assumed in the above subsections, respectively. It can be seen from Fig. 4 that the $\Pi/\Pi_{Am}-\psi$ curve is surrounded by the $\Pi_A/\Pi_{Am}-\psi_A$ curve. The maximum heating load, the COP at the heating load Π_m, and the COP for a given heating load increase after optimizing the heat surface transfer area distributions. These show that the cycle performance is improved with the optimal heat transfer area distributions. For example, the maximum heating load ratio is $\Pi_m/\Pi_{Am}=0.65$, which shows that the maximum heating load increases 54% after the heat transfer surface area distributions are optimized; the COP at the maximum heating load Π_m is 1.0 before optimizing the heat transfer surface area distributions, the COP at the same heating load Π_m is 1.25 after optimizing the heat transfer surface area distributions, which shows that the COP increases 25% with the optimal heat transfer surface area distribution.

5. Discussions

5.1. Optimal performance for fixed total heat inventory

The performance optimization can also be carried out by assuming that the total heat inventory is fixed [24,25]. Using $UA=U_1A_1+U_2A_2+U_3A_3+U_4A_4$ to replace $A=A_1+A_2+A_3+A_4$, i.e. using the distribution of the heat inventory to replace the distribution of the heat-transfer surface area, according to Eq. (12), one can obtain the fundamental optimal relation between the COP ψ_{UA} and the heating load Π_{UA} for fixed total heat inventory with linear phenomenological heat transfer law

$$\Pi_{UA}=(1+\xi)UA\times\frac{(1+\xi)(T_e^{-1}-T_g^{-1})+[T_a^{-1}+\xi T_c^{-1}-(1+\xi)T_e^{-1}]\psi_{UA}}{4(1+\xi)^2\psi_{UA}} \tag{16}$$

According to Eq. (13), one can obtain the relation between the optimal temperatures T_{1UA}, T_{2UA}, T_{3UA}, and T_{4UA} of working substance and the optimal COP for fixed total heat inventory with linear phenomenological heat transfer law

$$T_{1UA}=\frac{T_g}{1+b_1T_gu_{UA}},\qquad T_{2UA}=\frac{T_a}{1-b_2T_au_{UA}}$$
$$T_{3UA}=\frac{T_c}{1-b_3T_cu_{UA}},\qquad T_{4UA}=\frac{T_e}{1+T_eu_{UA}} \tag{17}$$

where

$$u_{UA}=\frac{[T_a^{-1}+\xi T_c^{-1}-(1+\xi)T_e^{-1}]\psi_{UA}-(1+\xi)(T_g^{-1}-T_e^{-1})}{2(1+\xi)\psi_{UA}}$$

According to Eq. (14), one can obtain the relations between the optimal heat inventory distributions and the optimal COP

$$\frac{U_1A_1}{UA}=\frac{1}{2\psi_{UA}},\qquad \frac{U_2A_2}{UA}=\frac{1}{2(1+\xi)}$$
$$\frac{U_3A_3}{UA}=\frac{\xi}{2(1+\xi)},\qquad \frac{U_4A_4}{UA}=\frac{\psi_{UA}-1}{2\psi_{UA}} \tag{18}$$

According to Eq. (14), one can obtain that $U_1A_1 + U_4A_4 = U_2A_2 + U_3A_3 = \frac{1}{2}UA$. It shows that the total heat inventory should be distributed equally in the sides of heat input and heat output.

5.2. *Special cases*

The results of this paper includes the optimal performance of the endoreversible four-heat-reservoir absorption heat pump cycle, the endoreversible three-heat-reservoir heat pump cycle, and the endoreversible Carnot heat pump cycle.

When $U_2 = U_3$ and $T_a = T_c$, the endoreversible four-heat-reservoir absorption heat pump cycle becomes the endoreversible three-heat-reservoir heat pump cycle. According to Eq. (5), one can obtain the general relation between the COP and the heating load of an endoreversible three-heat-reservoir heat pump cycle as follow

$$\left[T_g^n - \Pi/(U_1A_1\psi)\right]^{-1/n} - \psi\left[T_a^n + \Pi/(U_2A_2)\right]^{-1/n} + (\psi - 1)\left[T_e^n - \Pi(\psi - 1)/(U_4A_4\psi)\right]^{-1/n} = 0 \quad (19)$$

According to Eq. (12), one can obtain the fundamental optimal relation between the COP and the heating load of an endoreversible three-heat-reservoir heat pump with linear phenomenological heat transfer law [19,20] as follow

$$\Pi_A = AU_4\psi_A \frac{T_e^{-1} - T_g^{-1} + (T_a^{-1} - T_e^{-1})\psi_A}{[1 - b_1 - (1 + b_2)\psi_A]^2} \quad (20)$$

When $U_2 = U_3$, $T_a = T_c$ and $T_g \to \infty$, the heat reservoir T_g is equivalent to a work reservoir. Therefore, the endoreversible four-heat-reservoir absorption heat pump cycle becomes the endoreversible Carnot heat pump cycle. According to Eq. (5), one can obtain the general relation between the COP and the heating load of an endoreversible Carnot heat pump cycle as follow

$$\psi\left[T_a^n + \Pi/(U_2A_2)\right]^{-1/n} - (\psi - 1)\left[T_e^n - \Pi(\psi - 1)/(U_4A_4\psi)\right]^{-1/n} = 0 \quad (21)$$

According to Eq. (12), one can obtain the fundamental optimal relation between the COP and the heating load of an endoreversible Carnot heat pump with linear phenomenological heat transfer law [26] as follow

$$\Pi_A = AU_2 \frac{T_e - (\psi_A - 1)T_a/\psi_A}{T_eT_a[(1 + \delta(\psi_A - 1)/\psi_A]^2} \quad (22)$$

where $\delta = (U_2/U_4)^{1/2}$.

6. Conclusion

The endoreversible four-heat-reservoir absorption heat pump cycle model with a generalized heat transfer law $Q \propto \Delta(T^n)$ is established. The general relation between the COP and the heating load is derived by using finite-time thermodynamics. The fundamental optimal relation between the COP and the heating load, the optimal temperatures of the working substance and the optimal heat transfer surface area distributions of the four heat exchangers of the endoreversible four-heat-reservoir absorption heat pump cycle with linear phenomenological heat transfer law are derived in this paper. Moreover, the effects of the heat transfer law on the performance of the cycle are studied by numerical examples. The performance comparison before and after optimizing the distribution of the total heat transfer surface area is performed by numerical example, and the results show that the performance is improved after the heat transfer surface area distributions are optimized. The results of this paper include the optimal performance of the endoreversible three-heat-reservoir heat pump cycle and the optimal performance of the endoreversible Carnot heat pump cycles. The results obtained herein can provide some theoretical guidance for the optimal design of absorption heat pumps. It is necessary to confirm the analysis results by using the experimental data in the future work.

References

[1] F.L. Curzon, B. Ahlborn, Efficiency of a Carnot engine at maximum power output, Amer. J. Phys. 43 (1) (1975) 22–24.
[2] B. Andresen, R.S. Berry, M.J. Ondrechen, P. Salamon, Thermodynamics for processes in finite time, Acc. Chem. Res. 17 (8) (1984) 266–271.
[3] S. Sieniutycz, P. Salamon, Advances in Thermodynamics, vol. 4: Finite Time Thermodynamics and Thermoeconomics, Taylor & Francis, New York, 1990.
[4] A. de Vos, Endoreversible Thermodynamics of Solar Energy Conversion, Oxford University Press, Oxford, 1992.
[5] A. Bejan, Entropy generation minimization: The new thermodynamics of finite-size devices and finite-time processes, J. Appl. Phys. 79 (3) (1996) 1191–1218.
[6] R.S. Berry, V.A. Kazakov, S. Sieniutycz, Z. Szwast, A.M. Tsirlin, Thermodynamic Optimization of Finite Time Processes, Wiley, Chichester, 1999.
[7] L. Chen, C. Wu, F. Sun, Finite time thermodynamic optimization or entropy generation minimization of energy systems, J. Non-Equilib. Thermodyn. 24 (4) (1999) 327–359.
[8] J.M. Gordon, K.C. Ng, Cool Thermodynamics, Cambridge Int. Science Publishers, Cambridge, 2000.
[9] L. Chen, F. Sun, Advances in Finite-Time Thermodynamics: Analysis and Optimization, Nova Science, New York, 2004.
[10] A. Durmayaz, O.S. Sogut, B. Sahin, H. Yavuz, Optimization of thermal systems based on finite-time thermodynamics and thermoeconomics, Progr. Energy Combust. Sci. 30 (2) (2004) 175–217.
[11] L. Chen, Finite-Time Thermodynamic Analysis of Irreversible Processes and Cycles, Higher Education Press, Beijing, 2005.
[12] J. Chen, Z. Yan, Equivalent combined systems of three-heat-reservoir heat pumps, J. Chem. Phys. 90 (9) (1989) 4951–4955.
[13] K.E. Herold, Performance predictions of absorption cycles using an endoreversible model, in: Proceedings of Int. Sorption Heat Pump Conference, Munich, 1999, pp. 1–6.
[14] S. Goktun, The effect of irreversibilities on the performance of a three-heat-source heat pump, J. Phys. D: Appl. Phys. 29 (20) (1996) 2823–2825.
[15] J. Chen, Optimal performance analysis of irreversible cycles used as heat pumps and refrigerators, J. Phys. D: Appl. Phys. 30 (4) (1997) 582–587.
[16] A. Kodal, B. Sahin, I. Ekmekci, T. Yilmaz, Thermoeconomic optimization for irreversible absorption refrigerators and heat pumps, Energy Convers. Mgmt. 44 (1) (2003) 109–123.
[17] J. Chen, The general performance characteristics of an irreversible absorption heat pump operating between four temperature levels, J. Phys. D: Appl. Phys. 32 (12) (1999) 1428–1433.
[18] L. Chen, X. Qin, F. Sun, C. Wu, Irreversible absorption heat pump and its optimal performance, Appl. Energy 81 (1) (2005) 55–71.
[19] L. Chen, C. Wu, F. Sun, Optimal coefficient of performance and heating load relationship of a three-heat-reservoir endoreversible heat pump, Energy Convers. Mgmt. 38 (8) (1997) 727–733.
[20] L. Chen, Y. Bi, C. Wu, F. Sun, Unified description of endoreversible cycles for linear phenomenological heat-transfer law, Int. J. Energy Environment Economics 9 (2) (1999) 77–93.

[21] H.B. Callen, Thermodynamics and an Introduction to Thermostatics, Wiley, New York, 1985.
[22] A. de Vos, Efficiency of some heat engines at maximum power conditions, Amer. J. Phys. 53 (6) (1985) 570–573.
[23] T. Zheng, L. Chen, F. Sun, C. Wu, The influence of heat resistance and heat leak on the performance of a four-heat-reservoir absorption refrigerator with heat transfer law of $Q \propto \Delta(T^{-1})$, Int. J. Thermal Sci. 43 (12) (2004) 1187–1195.
[24] A. Bejan, J.V.C. Vargas, Optimal allocation of a heat exchanger inventory in heat driven refrigerators, Int. J. Heat Mass Transfer 38 (16) (1995) 2997–3004.
[25] M. Sokolov, J.V.C. Vargas, A. Bejan, Thermodynamic optimization of solar-driven refrigerators, Trans. ASME, J. Sol. Energy Engrg. 118 (2) (1996) 130–135.
[26] F. Sun, W. Chen, L. Chen, C. Wu, Optimal performance of an endoreversible Carnot heat pump, Energy Convers. Mgmt. 38 (14) (1997) 1439–1443.